Ασκήσεις Αλυσιδωτή Παραγώγιση

1. Να βρεθούν οι παράγωγοι των ακόλουθων συναρτήσεων:
   a. $f(x) = (2x^2 + x + 4)^4$.
   b. $f(x) = \left(\frac{\sqrt{x}}{4} - 1\right)^{-3}$.
   c. $f(x) = \sqrt{x^2 + 4x + 6}$.
   d. $f(x) = \sin(\cos x)$.
   e. $f(x) = \tan(x^2 + 1)$.
   f. $f(x) = \frac{1}{\sin^2 x}$.
   g. $f(x) = \frac{1}{5+\frac{1}{x^2}}$

2. Να βρεθούν οι παράγωγοι των συναρτήσεων:
   a. $f(x) = x\cos\left(2\sqrt{x}\right) + 5$.
   b. $f(x) = x^2 \sin\frac{1}{x}$.
   c. $f(x) = \sqrt{2 + x\sin x}$.
   d. $f(x) = \sin\left(\frac{x}{\sqrt{x^2+1}}\right)$.

3. Να βρεθούν οι δεύτεροι παράγωγοι των συναρτήσεων:
   a. $f(x) = \sin(1 + x^2)$.
   b. $f(x) = e^{\sqrt{x}}$.
   c. $f(x) = \ln(x^3 + 1)$.
   d. $f(x) = \frac{1}{\cos^2 x}$.

Δρ. Ιωάννης Τσιμπερίδης
ΕΔΙΠ Τμ. Πληροφορικής ΔΠΘ

Λύσεις

1.

a. $f'(x) = 4(2x^2 + x + 4)^3(2x^2 + x + 4)' = 4(4x + 1)(2x^2 + x + 4)^3$

b. $f'(x) = -3\left(\frac{\sqrt{x}}{4} - 1\right)^{-4}\left(\frac{\sqrt{x}}{4} - 1\right)' = -\frac{3}{8}\frac{1}{\sqrt{x}}\left(\frac{\sqrt{x}}{4} - 1\right)^{-4}$

c. $f'(x) = \frac{1}{2}\frac{1}{\sqrt{x^2+4x+6}}(x^2 + 4x + 6)' = \frac{2x+4}{2\sqrt{x^2+4x+6}}$

d. $f'(x) = \cos(\cos x)\,(\cos x)' = -\sin x \cos(\cos x)$

e. $f'(x) = \frac{1}{\cos^2(x^2+1)}(x^2 + 1)' = \frac{2x}{\cos^2(x^2+1)}$

f. $f'(x) = -2\frac{1}{\sin^3 x}(\sin^2 x)' = -2\frac{2\sin x(\sin x)'}{\sin^3 x} = 4\frac{\cos x}{\sin^2 x}$

h. $f'(x) = -\frac{1}{\left(5+\frac{1}{x^2}\right)^2}\left(5 + \frac{1}{x^2}\right)' = -\frac{-2\frac{1}{x^3}}{\left(5+\frac{1}{x^2}\right)^2} = \frac{2}{x^3\left(5+\frac{1}{x^2}\right)^2}$

2.

a. $f'(x) = x\left(\cos\left(2\sqrt{x}\right)\right)' + \cos\left(2\sqrt{x}\right) = -x\sin\left(2\sqrt{x}\right)\left(2\sqrt{x}\right)' + \cos\left(2\sqrt{x}\right) = -\sqrt{x}\sin\left(2\sqrt{x}\right) + \cos\left(2\sqrt{x}\right)$.

b. $f'(x) = x^2\left(\sin\frac{1}{x}\right)' + 2x\sin\frac{1}{x} = x^2\cos\frac{1}{x}\left(\frac{1}{x}\right)' + 2x\sin\frac{1}{x} = -\cos\frac{1}{x} + 2x\sin\frac{1}{x}$.

c. $f'(x) = \frac{1}{2\sqrt{2+x\sin x}}(2 + x\sin x)' = \frac{x\cos x+\sin x}{2\sqrt{2+x\sin x}}$.

d. $f'(x) = \cos\left(\frac{x}{\sqrt{x^2+1}}\right)\left(\frac{x}{\sqrt{x^2+1}}\right)' = \cos\left(\frac{x}{\sqrt{x^2+1}}\right)\frac{\sqrt{x^2+1}-x\left(\sqrt{x^2+1}\right)'}{x^2+1} = .\cos\left(\frac{x}{\sqrt{x^2+1}}\right)\frac{\sqrt{x^2+1}-x\frac{1}{2\sqrt{x^2+1}}\left(x^2+1\right)'}{x^2+1} = \cos\left(\frac{x}{\sqrt{x^2+1}}\right)\frac{\sqrt{x^2+1}-\frac{x^2}{\sqrt{x^2+1}}}{x^2+1} = \cos\left(\frac{x}{\sqrt{x^2+1}}\right)\frac{1}{\sqrt{(x^2+1)^3}}$

3.

a. $f'(x) = (\sin(1 + x^2))' = (1 + x^2)' = 2x\cos(1 + x^2)$.

$f''(x) = (2x\cos(1 + x^2))' = 2x(\cos(1 + x^2))' + (2x)'\cos(1 + x^2)$

$= -2x\sin(1 + x^2)\,(1 + x^2)' + 2\cos(1 + x^2) = -4x^2\sin(1 + x^2) + 2\cos(1 + x^2)$

b. $f'(x) = \left(e^{\sqrt{x}}\right)' = e^{\sqrt{x}}\left(\sqrt{x}\right)' = \frac{e^{\sqrt{x}}}{2\sqrt{x}}$.

$$f''(x) = \frac{\left(e^{\sqrt{x}}\right)'2\sqrt{x} - e^{\sqrt{x}}\left(2\sqrt{x}\right)'}{4x} = \frac{\frac{e^{\sqrt{x}}}{2\sqrt{x}}2\sqrt{x} - e^{\sqrt{x}}\frac{1}{\sqrt{x}}}{4x} = \frac{e^{\sqrt{x}} - e^{\sqrt{x}}\frac{1}{\sqrt{x}}}{4x} = e^{\sqrt{x}}\frac{\sqrt{x}-1}{4x\sqrt{x}}$$

c. $f'(x) = (\ln(x^3 + 1))' = \frac{3x^2}{x^3+1}$.

Δρ. Ιωάννης Τσιμπερίδης
ΕΔΙΠ Τμ. Πληροφορικής ΔΠΘ

$$f''(x) = \left(\frac{3x^2}{x^3+1}\right)' = \frac{(3x^2)'(x^3+1) - 3x^2(x^3+1)'}{(x^3+1)^2} = \frac{6x(x^3+1) - 9x^4}{(x^3+1)^2} = 3x\frac{2-x^3}{(x^3+1)^2}$$

d. $f'(x) = \left(\frac{1}{\cos^2 x}\right)' = -\frac{1}{\cos^4 x}(\cos^2 x)' = -\frac{2\cos x \sin x}{\cos^4 x} = -\frac{2\sin x}{\cos^3 x}$

$$f''(x) = \left(-\frac{2\sin x}{\cos^3 x}\right)' = -\frac{2\cos x \cos^3 x - 2\sin x \cdot 3\cos^2 x\,(-\sin x)}{\cos^6 x} = -\frac{2\cos^2 x - 6\sin^2 x}{\cos^6 x}$$