Τριγωνομετρικές Ασκήσεις

1. Να υπολογιστούν τα:
   a. $\sin\frac{7\pi}{12}$.
   b. $\cos\frac{5\pi}{12}$.
   c. $\cos\frac{7\pi}{12}$.
   d. $\sin\frac{5\pi}{12}$
   e. $\sin\frac{7\pi}{24}$.
   f. $\cos\frac{5\pi}{24}$.

2. Να αποδειχτεί ότι:
   a. $\sin(\pi - \theta) = \sin\theta$
   b. $\cos(\pi - \theta) = -\cos\theta$

3. Να λυθούν οι τριγωνομετρικές εξισώσεις:
   a. $2\cos^2 x = \sin x + 1$
   b. $2\sin x \cos x - 1 = 2\cos x - \sin x$
   c. $\tan^4 x = \frac{2}{\cos^2 x} + 1$
   d. $\cos(2x) + \sin(x) = 0$
   e. $\sin(2x) + \sin(x) = 0$
   f. $\sin(2x) - \sqrt{3}\cos(x) = 0$

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Λύσεις

1.

a. $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \sin\frac{\pi}{3}\cos\frac{\pi}{4} = \frac{\sqrt{2}}{2}\frac{1}{2} + \frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$

b. $\cos\frac{5\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{\pi}{6}\right) = \cos\frac{\pi}{4}\cos\frac{\pi}{6} - \sin\frac{\pi}{4}\sin\frac{\pi}{6} = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}-\sqrt{2}}{4}$

c. $\cos\frac{7\pi}{12} = \pm\sqrt{1 - \left(\frac{\sqrt{6}+\sqrt{2}}{4}\right)^2} = -\frac{\sqrt{2-\sqrt{3}}}{2}$

d. $\sin\frac{5\pi}{12} = \pm\sqrt{1 - \left(\frac{\sqrt{6}-\sqrt{2}}{4}\right)^2} = \frac{\sqrt{2+\sqrt{3}}}{2}$

e. $\sin\frac{7\pi}{24} = \sqrt{\frac{1-\cos\frac{7\pi}{12}}{2}} = \sqrt{\frac{1+\frac{\sqrt{2-\sqrt{3}}}{2}}{2}} = \frac{\sqrt{2+\sqrt{2-\sqrt{3}}}}{2}$

f. $\cos\frac{5\pi}{24} = \sqrt{\frac{1+\cos\frac{5\pi}{12}}{2}} = \sqrt{\frac{1+\frac{\sqrt{6}-\sqrt{2}}{4}}{2}}$

2.

a. $\sin(\pi - \theta) = \sin\pi\cos(-\theta) + \sin(-\theta)\cos\pi = -\sin(-\theta) = \sin\theta$

b. $\cos(\pi - \theta) = \cos\pi\cos(-\theta) - \sin\pi\sin(-\theta) = -\cos(-\theta) = -\cos\theta$

3.

a. $2\cos^2 x = \sin x + 1 \overset{\square}{\Rightarrow} 2(1 - \sin^2 x) = \sin x + 1 \overset{\square}{\Rightarrow} 2 - 2\sin^2 x = \sin x + 1 \overset{\square}{\Rightarrow} 2\sin^2 x + \sin x - 1 = 0$

$\Delta = 9$

$\sin x_{1,2} = \frac{-1 \pm 3}{4} \overset{\square}{\Rightarrow} \sin x_1 = \frac{1}{2} \quad \square \quad \sin x_2 = -1$

$x = 2k\pi + \frac{\pi}{6} \quad \square \quad x = 2k\pi + \pi - \frac{\pi}{6} \quad \square \quad x = 2k\pi - \frac{\pi}{2}$

b. $2\sin x\cos x - 1 = 2\cos x - \sin x \overset{\square}{\Rightarrow} 2\sin x\cos x - 2\cos x + \sin x - 1 = 0 \overset{\square}{\Rightarrow}$

$2\cos x(\sin x - 1) + \sin x - 1 = 0 \overset{\square}{\Rightarrow} (2\cos x + 1)(\sin x - 1) = 0$

$2\cos x + 1 = 0 \quad \square \quad \sin x - 1 = 0$

$\cos x = -\frac{1}{2} \quad \square \quad \sin x = 1$

$x = 2k\pi + \frac{2\pi}{3} \quad \square \quad x = 2k\pi + \frac{\pi}{2}$

c. $\tan^4 x = \frac{2}{\cos^2 x} + 1 \overset{\square}{\Rightarrow} \frac{\sin^4 x}{\cos^4 x} = \frac{2}{\cos^2 x} + 1 \overset{\square}{\Rightarrow} \sin^4 x = 2\cos^2 x + \cos^4 x \overset{\square}{\Rightarrow} (1 - \cos^2 x)^2 = 2\cos^2 x + \cos^4 x \overset{\square}{\Rightarrow}$

$1 - 2\cos^2 x + \cos^4 x = 2\cos^2 x + \cos^4 x \overset{\square}{\Rightarrow} 4\cos^2 x = 1 \overset{\square}{\Rightarrow} \cos x = \pm\frac{1}{2}$

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$$x = k\pi \pm \frac{\pi}{3}$$

d.

$$\cos(2x) = 1 - 2\sin^2(x)$$

Άρα:

$$1 - 2\sin^2(x) + \sin(x) = 0 \overset{\square}{\Rightarrow} 2\sin^2(x) - \sin(x) - 1 = 0$$
$$y = \sin(x)$$

Συνεπώς:

$$2y^2 - y - 1 = 0$$
$$\Delta = (-1)^2 - 4 \cdot 2 \cdot (-1) = 9$$
$$y_{1,2} = \frac{1 \pm \sqrt{9}}{4} \Rightarrow y_1 = 1 \quad \text{και} \quad y_2 = -\frac{1}{2}$$
$$x_1 = \frac{\pi}{2} + 2k\pi \quad \text{και} \quad x_2 = -\frac{\pi}{6} + 2k\pi$$

e.

$$\sin(2x) = 2\sin(x)\cos(x)$$

Άρα:

$$2\sin(x)\cos(x) + \sin(x) = 0 \overset{\square}{\Rightarrow} \sin(x)\,(2\cos(x) + 1) = 0$$

Επομένω2:

$$\sin(x) = 0 \quad \text{ή} \quad 2\cos(x) + 1 = 0$$
$$\sin(x) = 0 \quad \text{ή} \quad \cos(x) = -\frac{1}{2}$$
$$x = k\pi \quad \text{ή} \quad x = \frac{2\pi}{3} + 2k\pi \quad \text{ή} \quad x = \frac{4\pi}{3} + 2k\pi$$

f.

$$\sin(2x) = 2\sin(x)\cos(x)$$

Άρα:

$$2\sin(x)\cos(x) - \sqrt{3}\cos(x) = 0 \overset{\square}{\Rightarrow} \cos(x)\,(2\sin(x) - \sqrt{3}) = 0$$

Επομένως:

$$\cos(x) = 0 \quad \text{ή} \quad 2\sin(x) - \sqrt{3} = 0$$
$$\cos(x) = 0 \quad \text{ή} \quad \sin(x) = \frac{\sqrt{3}}{2}$$
$$x = k\pi + \frac{\pi}{2} \quad \text{ή} \quad x = \frac{\pi}{3} + 2k\pi \quad \text{ή} \quad x = \frac{2\pi}{3} + 2k\pi$$

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