

**Ασκήσεις Διπλά Ολοκληρώματα
Ορθογώνια Χωρία**

Να υπολογιστούν τα διπλά ολοκληρώματα των συναρτήσεων:

1. $f(x, y) = 1 - 6x^2y$ για $0 \leq x \leq 2$ και $-1 \leq y \leq 1$
2. $f(x, y) = xy(1 - 3y)$ για $0 \leq x \leq 2$ και $1 \leq y \leq 2$
3. $f(x, y) = e^y \cos x$ για $0 \leq x \leq \pi/2$ και $0 \leq y \leq 1$
4. $f(x, y) = x \sin(xy)$ για $0 \leq x \leq \pi/2$ και $1 \leq y \leq 2$
5. $f(x, y) = e^{2x+y}$ για $0 \leq x \leq 1$ και $0 \leq y \leq 1$
6. $f(x, y) = \frac{xy}{\sqrt{x^2+y^2}}$ για $0 \leq x \leq 1$ και $1 \leq y \leq 2$

Λύσεις

$$1. \int_{-1}^1 \int_0^2 (1 - 6x^2y) dx dy = \int_{-1}^1 [x - 2x^3y]_0^2 dy = \int (2 - 16y) dy = [2y - 8y^2]_{-1}^1 = 4$$

$$2. \int_1^2 \int_0^2 xy(1 - 3y) dx dy = \int_1^2 \int_0^2 (xy - 3xy^2) dx dy = \int_1^2 \left[\frac{1}{2}x^2y - \frac{3}{2}x^2y^2 \right]_0^2 dy = \int_1^2 (2y - 6y^2) dy = [y^2 - 2y^3]_1^2 = -11$$

$$3. \int_0^1 \int_0^{\pi/2} e^y \cos x dx dy = \int_0^1 [e^y \sin x]_0^{\pi/2} dy = \int_0^1 e^y dy = [e^y]_0^1 = e - 1$$

$$4. \int_0^{\pi/2} \int_1^2 x \sin(xy) dy dx = \int_0^{\pi/2} \left[-x \frac{1}{x} \cos(xy) \right]_1^2 dx = \int_0^{\pi/2} (-\cos 2x + \cos x) dx = \left[-\frac{1}{2} \sin 2x + \sin x \right]_0^{\pi/2} = 1$$

$$5. \int_0^1 \int_0^1 e^{2x+y} dx dy = \int_0^1 \left[\frac{1}{2} e^{2x+y} \right]_0^1 dy = \frac{1}{2} \int_0^1 (e^{y+2} - e^y) dy = \frac{1}{2} [e^{y+2} - e^y]_0^1 = \frac{1}{2} (e^3 - e^2 - e + 1)$$

$$6. \int_1^2 \int_0^1 \frac{xy}{\sqrt{x^2+y^2}} dx dy = \int_1^2 \left[y \sqrt{x^2+y^2} \right]_0^1 dy = \int_1^2 (y \sqrt{y^2+1} - y^2) dy = \left[\frac{1}{3} \sqrt{y^2+1}^3 - \frac{y^3}{3} \right]_1^2 = \frac{1}{3} (5\sqrt{5} - 2\sqrt{2} - 7)$$